Statistical inference based on record data from Pareto model

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In this article, based on record values from the two-parameter Pareto distribution, maximum likelihood and Bayes estimators for the two unknown parameters are obtained. The Bayes estimates are obtained on the basis of the squared error loss and linear-exponential loss functions. The admissibility of some estimators is discussed. The problem of predicting the future record values, either point or interval prediction, from the Pareto distribution, based on the past record values observed, is considered from a Bayesian approach. Also, the maximum likelihood prediction of the future records and other classical methods are used for obtaining prediction intervals for the future records. Numerical computations are given for empirical comparison purposes.

Keywords: Admissibility; Bayes estimation; Bayes prediction; Conditional median prediction; Linear-exponential loss; Maximum likelihood prediction; Squared error loss

1. Introduction

Let \( \{X_i, i \geq 1\} \) be a sequence of independent and identically distributed (iid) continuous random variables, each distributed according to cumulative distribution function (cdf) \( F(t; \theta) \) and probability density function (pdf) \( f(t; \theta) \), where \( \theta \) is a vector of parameters. An observation \( X_j \) will be called an upper record value if its value exceeds that of all previous observations. Thus, \( X_j \) is an upper record if \( X_j > X_i \) for every \( i < j \). An analogous definition can be given for lower record values. Then the record times sequence \( \{T_n, n \geq 1\} \) is defined in the following manner:

\[
T_1 = 1, \quad \text{with probability 1,}
\]

and, for \( n \geq 2 \),

\[
T_n = \min\{j : j > T_{n-1}, X_j > X_{T_{n-1}}\}.
\]

The sequence of upper record values is defined by \( R_n = X_{T_n}, n = 1, 2, 3, \ldots \).